

# **ANALYSIS OF NICKEL-CADMIUM BATTERY RELIABILITY DATA CONTAINING ZERO FAILURES**

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### **INTRODUCTION**

This paper summarizes an effort by Gates Aerospace and the Reliability Analysis Center (RAC) to analyze reliability data on NiCd batteries used on various spacecraft. This data has been collected by Gates and represents a substantial reliability database from which 183 satellites have been in operation from between .1 and 22 years each, for a total of 278 million cell-hours of operation, with no failures to date. The survival time data for each satellite, which has been extracted from Ref. 3, is included in Table 1.

There are two primary concerns when addressing the reliability of parts or systems; 1) the reliability during the useful life and 2) the lifetime. It is possible with the data collected thus far to draw limited conclusions regarding both of these concerns. However, since the data contains no failures, the quantification of accurate failure rates or lifetimes cannot be made. The appropriate analysis methodology to use under these conditions is the use of confidence limits. By the use of this methodology, an upper bound (or worst case value) on failure rate and a lower bound on lifetime can be made.

To accomplish this, the methodology proposed by Nelson (Ref. 2) to attach confidence limits to the Weibull distribution has been used. Advantages of the Weibull distribution are that it has a sound theoretical basis in reliability theory, it is flexible in that it can approximate many distribution shapes, and solutions from it can be obtained in closed form without the calculation of integrals necessary for the normal and lognormal distributions.

TABLE 1: NiCd SURVIVAL TIMES (IN YEARS)

1.25	9.1	2.8	1.7
4.5	9.1	7.9	1.5
2	8.4	2.5	1.5
20	8.4	7.8	1.3
.9	8.2	4.2	3.4
2.4	7	2.2	1
2.8	3	2.2	.8
22	7.7	2.2	.8
17.8	7.6	2.1	1.9
2	.6	2	1.7
.2	5.5	4	1.6
.9	7.4	6.3	.5
7.2	7.2	2.1	1.6
4.8	6	1.6	1.4
1.75	7.2	6.8	1.4
1.4	.3	6.8	1.3
.7	12.2	6.6	1.3
.8	11.2	3	1.2
14.6	12.2	1	1.2
11.5	2.8	1	1.1
13.6	6.7	1	1.1
14	6	6.3	1
5.75	4.4	5	.9
2.2	9.8	2.6	.9
10.6	9.7	6.2	.8
7.25	1	.8	.6
10.2	2.8	.8	.5
9.8	4.8	.8	.5
2.6	10.1	.5	.5
5	4.5	.6	.4
9	3.6	.2	.4
9.8	8.6	.2	.4
10.8	9.7	4.5	.4
10.8	4	6	.2
5	4.1	4.3	.2
9.5	4	4.3	.2
10.2	9.1	5.2	.2
4.5	3.85	5.2	.1
3	3.5	4	.1
9.9	8.9	3.1	
.4	.8	4.1	
5.6	3.2	1	
1	8.5	2.2	
9.6	3.96	2.2	
4	8.3	2.2	
1	8.3	3.2	
3	2.11	3	
3	8.2	.8	
8.5	2.8	1.8	

The analysis being accomplished in the report is based on the entire battery and not individual cells. The reason for this is that the data was collected at the battery level and not the cell level. It is conceivable that individual cells could have failed and not been observed at the system level (Ref. 1). In this manner, for the purposes of this analysis, cell redundancy can be disregarded since the data is at the next higher level of assembly.

## BACKGROUND

The probability density function  $f(t)$  of the Weibull time to failure distribution is;

$$f(t) = \frac{\beta}{\alpha} \left(\frac{t}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

where

- $\alpha$  = characteristic life, time to 63% population failure
- $\beta$  = Weibull shape parameter
- $t$  = time

The reliability (probability of survival to a time  $t$ ) is;

$$R(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

And the hazard rate  $h(t)$  (or instantaneous failure rate), given the part has survived until time  $t$  is;

$$h(t) = \left(\frac{\beta}{\alpha}\right) \left(\frac{t}{\alpha}\right)^{\beta-1}$$

To estimate the value of the characteristic life in the Weibull distribution, the following maximum likelihood estimator is typically used;

$$\alpha = \left[ \sum_{i=1}^n T_i^\beta / r \right]^{\frac{1}{\beta}}$$

where

- $T_i$  = Time to fail of the  $i^{\text{th}}$  part or survival time of the  $i^{\text{th}}$  part if it has not failed
- $r$  = Number of failures
- $n$  = Total population of parts

Since the data collected and presented in Table 1 indicates  $r=0$ , a characteristic life of infinity implied by this estimate is clearly erroneous. The fact that no failures have been observed indicates only that enough time has not elapsed to experience failures. As stated previously, the appropriate analysis methodology to use under these conditions is to apply confidence limits to derive worst case reliability values. From this, lower bound estimates of lifetimes can be made within a given confidence level. To accomplish this, the Chi-square distribution can be utilized. The lower confidence limit for the Weibull distribution in the case where no failures have occurred is;

$$\alpha = \left[ 2 \sum_{i=1}^n T_i^\beta / \chi^2(C; 2r + 2) \right]^{\frac{1}{\beta}}$$

where

- $\chi^2$  = the chi-square percentile at C% confidence and  $r$  failures

### DATA ANALYSIS

This value of characteristic life was then calculated from the data for various values of beta and various confidence levels. Various beta values were used in this

calculation to allow interpretation of this data in the event that a specific beta value is determined in the future. Since time-to-failure data is not available, empirical betas cannot be determined. Typical beta values have been derived from Weibull analysis from similar NiCd cells and will be presented later in this paper.

The sum of the individual survival times raised to the power beta, as a function of beta, are as follows;

$\beta$	$\sum_{i=1}^n T_i^\beta$
1	793.02
2	6360.9
3	69923.6
4	948534
5	15012172
6	$2.64 \times 10^8$

The values of the Chi-square percentiles, taken from Chi-square tables are;

C (Confidence Level)	Chi-Square Percentile
.25	.5754
.50	1.386
.75	2.773
.90	4.605
.95	5.991
.975	7.378
.990	9.210
.995	10.60
.999	13.82

The resulting characteristic life estimates, as a function of beta and confidence level are given in Table 2. It is important to note here that the values in this table have been derived by assuming that the population of batteries from which the survival data was taken could exhibit the  $\beta$  values listed. For example, the  $\alpha$  lower

limit of 52 at 90% confidence is only valid if a  $\beta$  of 2 represented the original population.

TABLE 2: LOWER LIMIT OF  $\alpha$  (YEARS)

$\beta$	C								
	.25	.50	.75	.90	.95	.975	.990	.995	.999
1	2756	1144	572	344	265	215	172	150	115
2	148	96	67	52	46	42	37	35	30
3	62	47	37	31	29	27	25	24	22
4	43	34	29	25	24	23	21	21	19
5	35	29	26	23	22	21	20	20	19
6	31	27	24	22	21	20	19	19	18

While the values of  $\alpha$  in Table 2 may appear to be unrealistically high for low confidence levels and low  $\beta$  values, they are included to illustrate the dependency of characteristic life to these values.

The characteristic lives listed in Table 2 are therefore the lower confidence limit of the actual failure distribution. For example, assuming a beta of 4, one can be 90% certain that the characteristic life is greater than 25 years, or 99.9% certain it is greater than 19 years.

To estimate typical  $\beta$  values for NiCd batteries that can be used as estimates for this analysis, time to failure information contained in reference 1 was analyzed for cells made by four different manufacturers. Table 3 contains the results of Weibull plots from which the  $\alpha$  and  $\beta$  values were derived. From this data, the range of  $\beta$  values were observed to be between 1 and 4. Although the characteristic life  $\alpha$  from this data is a function of cycles, its absolute value is not important for the purposes of this analysis since extraction of typical  $\beta$  values was the primary concern. Based on this information, a conservative  $\beta$  value of 4 can be used in lieu of empirical  $\beta$ 's for Gates NiCd cells. If time-to-failure data becomes available, the chosen value of  $\beta$  can be modified.

TABLE 3: WEIBULL ANALYSIS RESULTS

Manufacturer	$\alpha$	$\beta$
1	120	4
2	50	4
3	170	2.3
4	80	1.0

While the characteristic life may be very high, of more interest may be the time to .1% or 1% cumulative failure, which will be much less than  $\alpha$ . If it is desired to calculate the time (t) to the P percentile failure of the population, the following can be used;

$$t = \alpha \left[ -\ln \left( 1 - \frac{P}{100} \right) \right]^{\frac{1}{\beta}}$$

If the characteristic life is the lower confidence limit as tabulated previously, the time to P percent failure will also be the lower confidence limit. Tables 4 and 5 present the lower limit of time to 1% and .1% cumulative population failure, respectively.

TABLE 4: LOWER LIMIT OF TIME TO 1% FAILURE

$\beta$	C								
	.25	.50	.75	.90	.95	.975	.990	.995	.999
1	27.7	11.5	5.75	3.46	2.66	2.16	1.73	1.51	1.16
2	14.8	9.62	6.72	5.21	4.61	4.21	3.71	3.51	3.01
3	13.4	10.1	7.98	6.70	6.26	5.83	5.40	5.18	4.75
4	13.6	10.7	9.18	7.92	7.60	7.28	6.65	6.65	6.02
5	13.9	11.6	10.4	9.16	8.77	8.37	7.97	7.97	7.57
6	14.4	12.5	11.2	10.2	9.75	9.29	8.83	8.83	8.36



TABLE 5: LOWER LIMIT OF TIME TO .1% FAILURE

$\beta$	C								
	.25	.50	.75	.90	.95	.975	.990	.995	.999
1	2.76	1.14	.57	.34	.26	.22	.17	.15	.12
2	4.68	3.04	2.12	1.64	1.45	1.33	1.17	1.11	.94
3	6.20	4.70	3.70	3.10	2.90	2.70	2.50	2.40	2.20
4	7.43	6.04	5.16	4.45	4.27	4.09	3.73	3.73	3.38
5	8.79	7.28	6.53	5.78	5.53	5.27	5.02	5.02	4.77
6	9.80	8.54	7.49	6.95	6.64	6.32	6.00	6.00	5.69

For example, using the characteristic life of 25 years for  $\beta = 4$  and 90% confidence, the worst case time (at 90% confidence) to reach 1% failure is;

$$t = 25 \left[ -\ln \left( 1 - \frac{1}{100} \right) \right]^{\frac{1}{4}} = 7.92 \text{ years}$$

In this example, there is 90% confidence that the time to 1% failure will be greater than 7.92 years.

### CONCLUSIONS

Survival data of NiCd batteries was analyzed to determine what, if any, conclusions could be drawn regarding the NiCd battery reliability or lifetime. Conventional techniques of using an exponential (constant failure rate) distribution with the Chi-square distribution to obtain confidence intervals of failure rate are of limited value since it addresses only the failure rate in the products useful life and does not address the product lifetime. Additionally, it is of limited value since it assumes a constant failure rate which is an erroneous assumption for NiCd cells.

In the case where survival data only is available (no observed failures) and where failure mechanisms are known to be wearout related, the use of the Weibull time to failure distribution can be used in conjunction with the Chi-square distribution (Ref. 2) to yield a lower limit of characteristic life (or time to a given percent failure). This estimate of the lower limit is a function of the confidence level of the characteristic life estimate and of the Weibull shape parameter,  $\beta$ . The

shape parameter  $\beta$  is a critical factor in lower bound life estimations and since the data used in this methodology is survival data only, the  $\beta$  cannot be derived from empirical data. Therefore to adequately use the analysis methodology outlined in this paper, a  $\beta$  value must either be known or derived from alternative means, such as life testing.

## REFERENCES

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3. "Orbital Performance History of Gates NiCD Aerospace Cells," Gates Aerospace Batteries Report, March 18, 1991.